Abstract

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Lecture 1: *Approximation on compact nowhere dense subsets of the complex plane.*

This lecture will begin with a description and answers to several approximation questions arising from early attempts to solve the invariant subspace problem for subnormal operators on a Hilbert space, and will conclude with the problem of extending the uniqueness property of the analytic functions to larger classes of functions defined on sets without interior points, a topic stretching back to 1892 and a confrontation between Borel and Poincaré.

Lecture 2. *Thomson's theorem and the semi-additivity of analytic capacity.*

In this lecture I shall present a strengthened version of Thomson's theorem on mean-square polynomial approximation based on a result of Tolsa to the effect that γ and γ^+ are equivalent capacities. As a corollary I will give the solution to a problem on polynomial approximation that first arose in connection with the invariant subspace problem for subnormal operators, and remained open from 1973 to 2010. I will also describe other applications to well-known theorems of Lavrentiev and Vitushkin on polynomial and rational approximation.

Lecture 3. *Absolutely continuous representing measures for $H^\infty(X)$.*

Let $H^\infty(X)$ denote the weak-$*$ closure of $R(X)$ in $L^\infty(X)$. In 1985 Khavinson showed that the usual Cauchy-Green formula

$$f(x_0) = \int_{\partial^* X} \frac{f^*(z)}{z - x_0}\, dz \qquad (*)$$

is valid for every $f \in H^\infty(X)$ and at every point x_0 in a Swiss cheese X where

$$\int_{\partial^* X} \frac{|dz|}{|z - x_0|} < \infty.$$

Here, $\partial^* X$ denotes the reduced boundary of X, and f^* is an appropriately defined boundary value for f on $\partial^* X$. In this lecture I will show that the representation formula $(*)$ remains valid at every non-peak point for $R(X)$, provided it is interpreted in a principal value sense. In addition, I will present an example to indicate in terms of Hausdorff dimension the possible sizes of a sets where the principal value integral exists, but formula $(*)$ fails. These results depend on a theorem of Davie concerning the bounded pointwise limits of analytic functions, Mel'nikov's peak point criterion for $R(X)$, and the semi-additivity of analytic capacity.